Paper Reference(s)

6679

Edexcel GCE

Mechanics M3

Advanced Subsidiary

Friday 6 June 2008 – Afternoon

Time: 1 hour 30 minutes

Materials required for examination	**Items included with question papers**
Mathematical Formulae (Green)	Nil

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulas stored in them.

Instructions to Candidates

In the boxes on the answer book, write the name of the examining body (Edexcel), your centre number, candidate number, the unit title (Mechanics M2), the paper reference (6678), your surname, other name and signature.
Whenever a numerical value of g is required, take g = 9.8 m s^{-2}.
When a calculator is used, the answer should be given to an appropriate degree of accuracy.

Information for Candidates

A booklet 'Mathematical Formulae and Statistical Tables' is provided.
Full marks may be obtained for answers to ALL questions.
There are 6 questions in this question paper.
The total mark for this paper is 75.

Advice to Candidates

You must ensure that your answers to parts of questions are clearly labelled.
You must show sufficient working to make your methods clear to the Examiner.
Answers without working may not gain full credit.

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1.

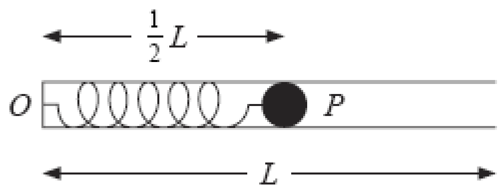

Figure 1

A light elastic spring, of natural length L and modulus of elasticity λ, has a particle P of mass m attached to one end. The other end of the spring is fixed to a point O on the closed end of a fixed smooth hollow tube of length L.

The tube is placed horizontally and P is held inside the tube with $OP = \frac{1}{2}L$, as shown in Figure 1. The particle P is released and passes through the open end of the tube with speed $\sqrt{(2gL)}$.

(*a*) Show that $\lambda = 8mg$.

(4)

The tube is now fixed vertically and P is held inside the tube with $OP = \frac{1}{2}L$ and P above O. The particle P is released and passes through the open top of the tube with speed u.

(*b*) Find u.

(5)

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2. A particle P moves with simple harmonic motion and comes to rest at two points A and B which are 0.24 m apart on a horizontal line. The time for P to travel from A to B is 1.5 s. The midpoint of AB is O. At time $t = 0$, P is moving through O, towards A, with speed u m s^{-1}.

(*a*) Find the value of u.

(4)

(*b*) Find the distance of P from B when $t = 2$ s.

(5)

(*c*) Find the speed of P when $t = 2$ s.

(2)

3.

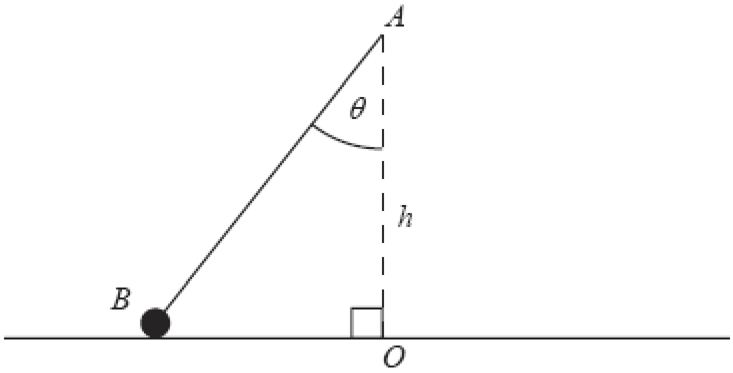

Figure 2

Figure 2 shows a particle B, of mass m, attached to one end of a light elastic string. The other end of the string is attached to a fixed point A, at a distance h vertically above a smooth horizontal table. The particle moves on the table in a horizontal circle with centre O, where O is vertically below A. The string makes a constant angle θ with the downward vertical and B moves with constant angular speed ω about OA.

(a) Show that $\omega^2 \leq \dfrac{g}{h}$.

(8)

The elastic string has natural length h and modulus of elasticity 2 mg.

Given that $\tan \theta = \dfrac{3}{4}$,

(b) find ω in terms of g and h.

(5)

4.

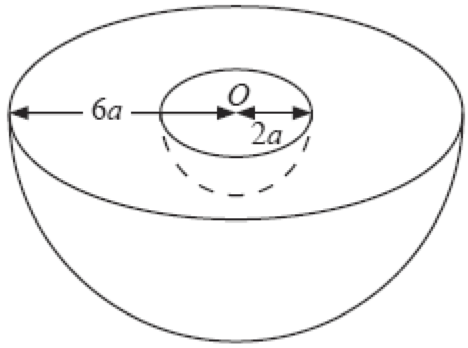

Figure 3

A uniform solid hemisphere, of radius $6a$ and centre O, has a solid hemisphere of radius $2a$, and centre O, removed to form a bowl B as shown in Figure 3.

(a) Show that the centre of mass of B is $\frac{30}{13}a$ from O.

(5)

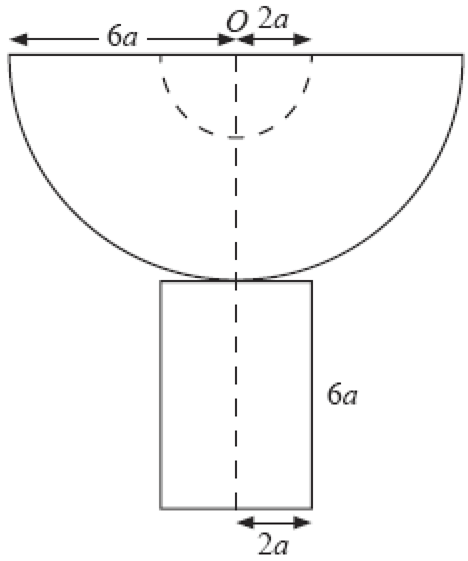

Figure 4

The bowl B is fixed to a plane face of a uniform solid cylinder made from the same material as B. The cylinder has radius $2a$ and height $6a$ and the combined solid S has an axis of symmetry which passes through O, as shown in Figure 4.

(b) Show that the centre of mass of S is $\frac{201}{61}a$ from O.

(4)

The plane surface of the cylindrical base of S is placed on a rough plane inclined at 12° to the horizontal. The plane is sufficiently rough to prevent slipping.

(c) Determine whether or not S will topple.

(4)

5. A particle P of mass m is attached to one end of a light inextensible string of length a. The other end of the string is attached to a fixed point O. The particle is released from rest with the string taut and OP horizontal.

(a) Find the tension in the string when OP makes an angle of 60° with the downward vertical.

(6)

A particle Q of mass $3m$ is at rest at a distance a vertically below O. When P strikes Q the particles join together and the combined particle of mass $4m$ starts to move in a vertical circle with initial speed u.

(b) Show that $u = \sqrt{\left(\frac{ga}{8}\right)}$.

(3)

The combined particle comes to instantaneous rest at A.

(c) Find

(i) the angle that the string makes with the downward vertical when the combined particle is at A,

(ii) the tension in the string when the combined particle is at A.

(6)

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6. A particle P of mass 0.5 kg moves along the positive x-axis. It moves away from the origin O under the action of a single force directed away from O. When $OP = x$ metres, the magnitude of the force is $\frac{3}{(x+1)^3}$ N and the speed of P is v m s^{-1}.

Initially P is at rest at O.

(a) Show that $v^2 = 6\left(1 - \frac{1}{(x+1)^2}\right)$.

(6)

(b) Show that the speed of P never reaches $\sqrt{6}$ m s^{-1}.

(1)

(c) Find x when P has been moving for 2 seconds.

(7)

TOTAL FOR PAPER: 75 MARKS

END

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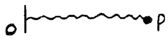
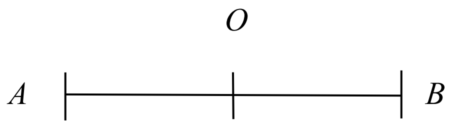
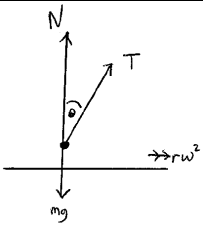
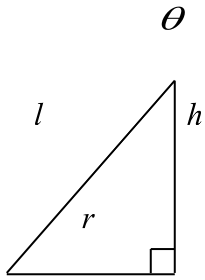
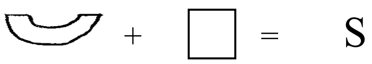
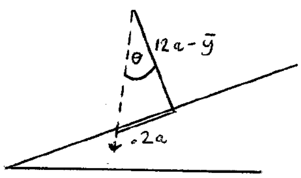
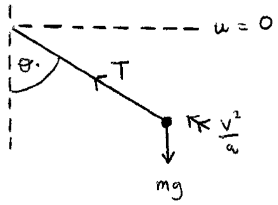
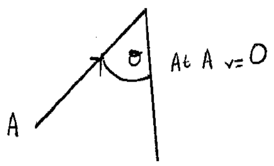
6679 Mechanics M3
Mark Scheme

Question Number	Scheme	Marks
Q1(a)	 EPE stored $= \frac{1}{2}\frac{\lambda}{L}\left(\frac{1}{2}L\right)^2 \quad \left(= \frac{\lambda L}{8}\right)$	B1
	KE gained $= \frac{1}{2}m\,2gL \quad (= mgL)$	B1
	EPE = KE $\Rightarrow \frac{\lambda L}{8} = \text{mg}\,L$ i.e. $\lambda = 8mg$*	M1A1cso
		(4)
(b)	EPE = GPE + KE	M1
	 $\frac{1}{2}\frac{8mg}{L}\left(\frac{1}{2}L\right)^2 = \frac{8mgL}{8} = mg\frac{L}{2} + \frac{1}{2}mu^2$	A1A1
	$\frac{mgL}{2} = \frac{m}{2}u^2 \quad \therefore u = \sqrt{gL}$	M1A1 **(5)**
		9 Marks

Question Number	Scheme	Marks
Q2 (a)		
	$T = 3 = \frac{2\pi}{\omega} \quad \therefore \ \omega = \frac{2\pi}{3}$	M1A1
	$u^2 = \omega^2\left(a^2 - x^2\right)$; $a = 0.12$, $u^2 = a^2\,\omega^2, u = 0.12 \times \omega$	M1
	$= 0.251 \text{ ms}^{-1}$ (0.25 m s^{-1})	A1 **(4)**
(b)	Time from $O \rightarrow A \rightarrow O = 1.5\text{s}$ $\quad \therefore t = 0.5$	B1
	$x = a \sin \omega t \quad \Rightarrow OP = 0.12 \sin\left(\frac{\pi}{3}\right)$	M1A1
	Distance from B is $0.12 - OP = 0.12 - 0.104\ldots = 0.016\text{m}$	M1A1 **(5)**
(c)	$v^2 = \omega^2\left(a^2 - x^2\right)$	M1
	$v = \frac{2\pi}{3}\sqrt{0.12^2 - 0.104....^2} = \frac{2\pi}{3} \times 0.0598 = 0.13 \text{ ms}^{-1}$	A1 **(2)**
		11 Marks

Question Number	Scheme	Marks
Q3 (a)		
	$\uparrow \quad T\cos\theta + N = Mg \quad$ **(1)**	M1A1
	$\rightarrow \quad T\sin\theta = mr\omega^2 \quad$ **(2)**	M1A1
		
	$\sin\theta = \frac{r}{l}$ from **(2)** $T = ml\omega^2$	
	sub into **(1)** $ml\cos\theta\,\omega^2 + N = mg$	M1
	$N = mg - mh\omega^2$	A1
	Since in contact with table $N \geq 0 \quad \therefore \omega^2 \leq \frac{g}{h}$ *	M1A1 cso
		(8)
(b)		
	$r:h:l = 3:4:5 \quad \therefore$ extension $= \frac{h}{4}$	B1
	$T = \frac{2mg}{h} \times \frac{h}{4} \quad = \frac{mg}{2}$	M1A1
	$T = ml\omega^2 = \frac{5mh}{4}\omega^2 \quad \omega = \sqrt{\frac{2g}{5h}}$	M1A1
		(5)
		13 marks

Question Number	Scheme	Marks
Q4 (a)		
	Mass $a^3 \frac{2}{3}\pi \times$: 216 8 208	M1A1
	27 1 26	
	C of M from O: $\frac{3}{8}\times 6a$ $\frac{3}{8}\times 2a$ $\bar{x}$ Use of $\frac{3}{8}r$	M1
	Moment : $216\times\frac{6a\times 3}{8} = 8\times\frac{2a\times 3}{8}+208\bar{x}$	M1
	$\bar{x} = \frac{480a}{208} = \frac{30a}{13}$ *	A1 cso **(5)**
(b)	 	
	Mass $\pi a^3 \times$: $\frac{416}{3}$ + 24 = $\frac{488}{3}$	B1
	C of M: $\frac{30}{13}a$ + $9a$ = $\bar{y}$	B1
	Moments : $320a + 216a = \frac{488}{3}\bar{y}$	M1
	$\bar{y} = \frac{201}{61}a$ *	A1 cso
(c)	 $12a - \frac{201}{61}a$	**(4)** M1
	$\tan\theta = \frac{2a}{12a-\frac{201}{61}a}$ $\tan\theta = \frac{2a}{....}$	M1
	$\theta = 12.93.....$	A1
	so critical angle = 12.93… $\therefore$ if $\theta = 12°$ it will <u>NOT</u> topple.	A1 ÷ **(4)** **13 marks**

Question Number	Scheme	Marks
Q5(a)	 Energy $\frac{1}{2}mv^2 = mga\cos\theta$ $v^2 = 2ga\cos\theta$	M1A1
	$F = ma$ ↖ $T - mg\cos\theta = \frac{mv^2}{a}$	M1A1
	Sub for $\frac{v^2}{a}$: $T = mg\cos\theta + 2mg\cos\theta$: $\theta = 60$ $\therefore T = \frac{3}{2}mg$	M1A1
		(6)
(b)	Speed of P before impact = $\sqrt{2ga}$	B1
	$\rightarrow\sqrt{2ga}$ $\rightarrow 0$ $\rightarrow u$ PCLM : • • $\longrightarrow$ • $\therefore u = \frac{\sqrt{2ga}}{4} = \sqrt{\frac{ga}{8}}$ * m 3m 4m	M1A1cso
		(3)
(c) (i)	 At A $v = 0$ so conservation of energy gives: $\frac{1}{2}4\cancel{m}u^2 = 4\cancel{m}\,ga\,(1-\cos\theta)$	M1A1
	$\frac{ga}{16} = ga\,(1-\cos\theta)$	M1
	$\cos\theta = \frac{15}{16}$, $\theta = 20^{\circ}$	A1
(ii)	↗ At A $T = 4mg\cos\theta = \frac{15mg}{4}$ (accept 3.75mg)	M1A1 **(6)**
		15 Marks

Question Number	Scheme	Marks
Q6 (a)	$F = ma\ (\rightarrow)\quad \dfrac{3}{(x+1)^3} = 0.5a = 0.5\ v\dfrac{dv}{dx}$	M1A1
	$\int \dfrac{3}{(x+1)^3}dx = 0.5\int v\ dv$ Separate and $\int$	M1
	$-\dfrac{3}{2(x+1)^2} = \dfrac{1}{4}v^2 (+\ c)$	A1
	$x = 0,\ v = 0 \Rightarrow c' = -\dfrac{3}{2} \quad \therefore \quad v^2 = 6\left(1 - \dfrac{1}{(x+1)^2}\right)$ *	M1A1 cso **(6)**
(b)	$\forall x \quad v^2 < 6 \quad \therefore \quad v < \sqrt{6} \quad (\because (x+1)^2$ always $> 0)$	B1 **(1)**
(c)	$v = \dfrac{dx}{dt} = \dfrac{\sqrt{6}\sqrt{(x+1)^2 - 1}}{x+1}$	M1
	$\int \dfrac{x+1}{\sqrt{(x+1)^2 - 1}}\ dx = \sqrt{6}\int dt$	M1
	$\sqrt{(x+1)^2 - 1} = \sqrt{6}\,t + c'$	M1 A1
	$t = 0,\ x = 0 \Rightarrow c' = 0$	M1
	$t = 2 \Rightarrow (x+1)^2 - 1 = (2\sqrt{6})^2$	M1
	$(x+1)^2 = 25 \quad \Rightarrow x = 4 \quad$ (c' need not have been found)	A1 cao
		(7)
		14 Marks